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Governing the green: game theory and mechanism design approaches to carbon compensation in professional golf

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Abstract. Professional golf tours face growing pressure to address their significant carbon footprint, which is primarily driven by extensive, season-long international travel. This paper develops a comprehensive framework to design an optimal, credible, and efficient carbon compensation mechanism for such sports organizations. This paper models the strategic interactions among key stakeholders, the Tour, sponsors, individual tournaments, players, and spectators, with a three-tiered cascade of principal-agent relationships. At the highest level, this paper analyzes the Tour-Sponsor dynamic as a repeated game where reputation for environmental commitment is a capital asset maintained under the threat of contract termination. At the intermediate level, this paper solves the Tour's problem of regulating its tournaments using a mechanism design approach with adverse selection, demonstrating that a non-linear menu of contracts is superior to uniform policies in managing heterogeneous abatement costs. Combining above two models, this paper creates an integrated model for the tour considering both the tournaments and the sponsors. Finally, at the local level, this paper shows how an individual tournament can efficiently delegate its emissions target to players and spectators via a simple internal carbon price. The findings provide a clear roadmap for sports leagues to implement structurally sound decarbonization strategies. The model's key predictions are also shown to be consistent with emerging empirical evidence from the world of professional golf.

Keywords: mechanism design, carbon compensation, game theory, environmental economics, sports sustainability

1. Introduction

In the global discourse on climate change, large-scale sporting events have increasingly come under scrutiny due to their extensive social influence, substantial resource consumption, and significant carbon emissions [1]. As a sport intrinsically linked to the natural environment, professional golf is no exception. The top-tier professional golf tours, such as the PGA Tour and DP World Tour, are facing unprecedented pressure to address their environmental impact [2]. In the context of global carbon neutrality goals, the carbon compensation strategies of these golf tours have evolved beyond an environmental responsibility into a core strategic challenge, one that intertwines brand reputation, commercial value, and future sustainability.

Golf competitions generate unique patterns of carbon footprint. Unlike other global events such as the Olympics or the FIFA World Cup, which are often centralized in a single city or region with significant

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infrastructure investments, golf tours have a "chronic" carbon footprint. The main contributor to their emissions is not the construction of new venues but the extensive travel required throughout the season. Golf tours typically involve continuous, global travel for players, sponsors, and fans, with air travel being the dominant source of emissions [1, 3].

Among the tours, spectator travel accounted for the largest portion, representing 45% of the total emissions, followed by player travel at 35%. In contrast, traditional operational sources such as energy consumption (4.70%), logistics and freight (3.46%), and waste management (0.29%) made up a much smaller share. This under scores that for golf tours, the primary focus for carbon reduction lies in transportation, particularly air travel. This highlights a crucial distinction: for golf tours, reducing carbon emissions necessitates a systemic, season-long strategy focusing on travel reductions, sustainable transportation alternatives, and efficient carbon offset mechanisms [2, 4, 5].

A golf *tour* refers to the entire series of events organized over a season, typically involving multiple tournaments held in different locations. In contrast, a *tournament* is a single competition within the broader tour, often associated with specific venues or locations. While the tour provides the overarching structure for the season, individual tournaments are the focal points where players compete for ranking points and prize money. Importantly, the relationship between the two is symbiotic, as each tournament's success contributes to the overall appeal and commercial value of the tour, while the tour's strategic decisions (such as scheduling, venue selection, and sponsorship partner-ships) influence the operational aspects of individual tournaments. In the context of carbon emissions, both the *tour* and the *tournament* are distinct entities with their own responsibilities but must collaborate to achieve sustainable outcomes.

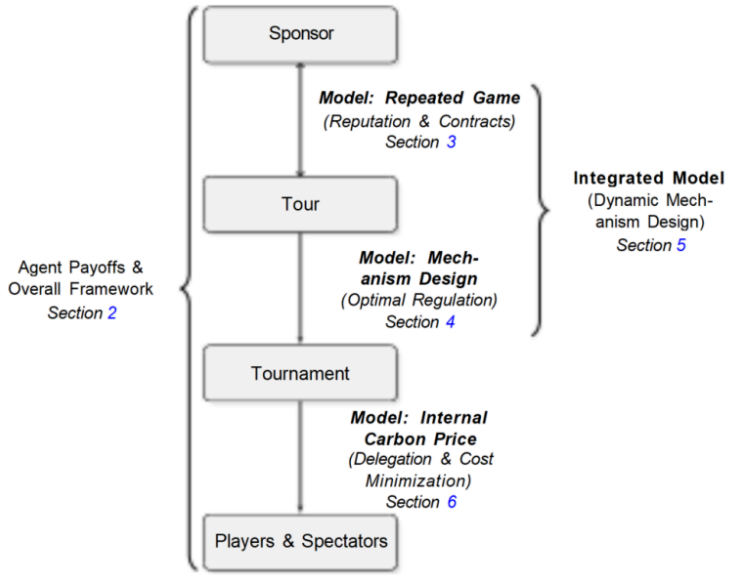


Figure 1. Principal-agent model for carbon compensation in professional golf

Beyond the operational challenges, the relationship between golf events and sponsors plays a pivotal role in shaping sustainability strategies. Sponsors, particularly those with their own stringent Environmental, Social, and Governance (ESG) goals, exert considerable influence on the strategies employed by these tours [2].

The overarching challenge facing these golf tours lies in the credibility of their carbon neutrality claims. For golf tours, it is not sufficient to announce carbon neutrality goals; these goals must be supported by

transparent, rigorous carbon accounting and high-quality, verifiable carbon offset projects [3]. Failure to do so risks accusations of "green washing", which can undermine a tour's brand value and commercial standing [6].

In this context, the design of an optimal carbon offset mechanism for large-scale golf tours becomes not just an environmental concern but a complex problem involving multiple stakeholders, including the tour organizers, tournament organizers, players, sponsors, and spectators. The application of game theory and mechanism design, complemented by multi-agent reinforcement learning, offers a robust framework for analyzing and optimizing the behavior of these stakeholders [7, 8].

This paper develops a multi-level theoretical framework with computer solving and simulations, to design an optimal carbon compensation mechanism for a professional golf tour. This paper models the problem as a three-tiered cascade of principal-agent relationships that mirrors the tour's governance structure. This hierarchical approach allows us to analyze how incentives and responsibilities are delegated downwards, from the highest strategic level of brand management to the on-the-ground implementation of emissions reduction. The overall structure of the structure of this paper and the models is illustrated in Figure 1.

By applying game theory and mechanism design, this paper provides a rigorous frame- work for analyzing and optimizing stakeholder behavior in the pursuit of carbon neutrality. This paper demonstrates why one-size-fits-all policies are inefficient and derive the properties of an optimal, incentive-compatible system. The model's predictions are further substantiated with recent empirical examples from professional golf, illustrating how reputation, con- tract menus, and internal pricing are already shaping the sport's response to the climate challenge.

2. Model framework

To analyze the complex challenge of carbon compensation in professional golf, this paper develops a multi-level model that deconstructs the problem into a series of interconnected strategic interactions. This hierarchical framework recognizes that the Tour operates simultaneously at different levels: (1) managing its long-term brand reputation with sponsors, (2) regulating the environmental performance of individual tournaments, and (3) indirectly influencing the behavior of players and spectators. The subsequent sections of this paper will analyze each level in turn, building from the top down.

2.1. A three-tiered governance structure

The model is organized around a three-tiered governance structure, with five primary agents whose decisions are nested within this hierarchy: (1) the Tour, (2) the individual Tournament, (3) the Sponsor, (4) the Player, and (5) the Spectator.

•Tier 1: Strategic reputation management (Tour-Sponsor) — At the highest level, the Tour acts as a long-term strategic player managing its brand and reputation to secure essential funding from its main Sponsor. The credibility of its environmental commitments is paramount. This paper models this dynamic as a repeated game where the Tour's "effort" to maintain its reputation is imperfectly observed by the Sponsor, who makes periodic renewal decisions. This top-level interaction, analyzed in Section 3, sets the overall incentive for the Tour to engage in meaningful climate action.

•Tier 2: Optimal regulation (Tour-Tournament) — To operationalize its environmental commitments, the Tour must act as a Principal, designing a regulatory mechanism to manage the emissions of its constituent Tournaments (the Agents). The key challenge here is asymmetric information: each Tournament has private knowledge of its own abatement costs. The Tour's problem, analyzed in Section 4, is to design an incentive-compatible mechanism that elicits this information and achieves emissions reductions efficiently.

•Tier 1 and 2 Synthesis: dynamic regulation as reputation management — While analyzing these tiers separately provides clear insights, the Tour's decisions are not made in a vacuum. A truly optimal strategy must connect the long-term reputational stakes from Tier 1 to the regulatory design in Tier 2. This paper therefore synthesizes these two levels in a unified dynamic mechanism design model, as detailed in Section 5. In this advanced framework, the Tour's choice of regulatory mechanism in each period becomes its primary action for managing its reputation, explicitly linking the cost of regulation to the value of its sponsorship agreements.

•Tier 3: Internal delegation (Tournament-Participants) — Once a Tournament is assigned an emissions target by the Tour's mechanism, it becomes a local Principal. It must then solve its own delegation problem: how to meet its target by inducing abatement from the primary emitters, the Players and Spectators. As this paper shows in Section 6, this can be achieved by implementing an internal carbon pricing scheme.

The central externality problem, i.e., the carbon footprint from travel by Players and Spectators, is thus managed through this cascade of principal-agent relationships. The reputational and financial consequences are passed up the chain, motivating each level to design mechanisms to control the level below it.

2.2. Payoff functions and decision variables

Each agent is assumed to be rational, seeking to maximize a payoff function that reflects their objectives within this structure.

The Tour (Π_{Tour}) The Tour's revenues are derived from global sponsors, fees from constituent tournaments, and media rights, which this paper omits in this model. Its costs include administration and any centralized carbon offsetting it undertakes (Equation (1)).

$$\begin{aligned}\Pi_{Tour} = (R_{s,global} + \textstyle\sum_{i=1}^{k}(F_{fixed,i} + F_{fee,i}(E_{T,i}))) \\ -(C_{op} + F_{tour}(\textstyle\sum_{i=1}^{k} E_{T,i}) + C_{abate,Tour})\end{aligned} \tag{1}$$

Here, is revenue from global sponsors. Each of the tournaments pays a fixed fee, $F_{fixed,i}$, and a variable carbon-related fee, $F_{fee,i}(E_{T,i})$, which is a function of that tournament's total emissions, $E_{T,i}$. This fee is a key mechanism design instrument for the Tour to influence tournament behavior. On the cost side, represents administrative expenses and operational costs, and is the cost of compensating for the emissions from all tournaments. The Tour's reputation, $\mathbb{R}_{Tour}$ is a decreasing function of the tour's total emissions, $E_{total} = \sum_{i=1}^{k} E_{T,i}$. It is the cost of centralized emission reduction efforts.

The Tournament ($\Pi_{Tournament}$) Each individual Tournament i acts as an agent of the Tour. It seeks to maximize its event-specific profit, subject to the regulations imposed upon it. Its total emissions, $E_{T,i}$, are the sum of emissions from all participating players (n_i) and attending spectators (m_i): $E_{T,i} = \sum_{j=1}^{n_i} e_{P,j} + \sum_{l=1}^{m_i} e_{S,l}$. A Tournament of type θ_i (representing its private abatement cost) that is assigned an emissions target E_i and transfer T_i by the Tour's mechanism earns a net utility of (Equation (2)):

$$\begin{aligned}\Pi_{T,i} = \ (R_{tickets,i} + \textstyle\sum_{j=1}^{n_i} F_{player,j}(e_{P,j}) + \sum_{l=1}^{m_i} F_{spectator,l}(e_{S,l})) \\ -(C_{prize,i} + F_{fixed,i} + F_{fee,i}(E_{T,i}) + C_{op,i} + C_{abate,i})\end{aligned} \tag{2}$$

$R_{tickets,i}$ is revenue from ticket sales. The sums and represent the total revenue from carbon fines levied on players and spectators, respectively. It is the prize purse, and includes operational costs, which may contain expenses related to on-site emission reduction efforts. The cost function also includes $C_{abate,i}$, the direct investment in on-site emission reduction activities. Specially, $C_{abate,i} = C(E_i, \theta_i)$, where E_i is the emissions target assigned to the tournament by the Tour's mechanism, and θ_i is the tournament's private abatement cost parameter.

The Sponsor ($\Pi_{Sponsor}$) Sponsors, whether at the Tour or Tournament level, seek to maximize the return on their investment. This return is composed of the commercial value from brand exposure and the monetized

value of ESG-related reputational gains (Equation (3)).

$$\Pi_{Sponsor} = (R_{ad}(R) + V_{ESG}(R)) - (F_{sponsor} + C_{activation}) \tag{3}$$

Here, is the advertising revenue generated, which is a function of the reputation (R) of the sponsored entity (R_{Tour} or $R_{T,i}$). Crucially, represents the tangible financial benefits of associating with a brand that has a strong environmental reputation, which can improve the sponsor's own stock valuation and consumer loyalty. It is the direct sponsorship fee, and represents ancillary marketing costs

It is worth noting that sponsorship contracts often involve significant entry and exit costs, i.e., switching costs, creating relationship inertia. This suggests that while sponsors may tolerate minor, short-term reputational fluctuations, they will react strongly to persistent or severe reputational damage, making the long-term credibility of the Tour's environmental strategy paramount.

The Player (U_{Player}) Players maximize a utility function comprising income and personal brand image, net of costs (Equation (4)).

$$U_{Player} = C_{prize} - C_{abate} \tag{4}$$

C_{prize} is prize money. Costs include standard travel expenses that this paper omits here, and the monetary or convenience cost of emission abatement, such as choosing lower-emission travel options or paying a carbon levy $F_{Player}(ep)$.

The Spectator ($U_{Spectator}$) Spectators maximize the utility derived from the event experience, balanced against their costs (Equation (5)).

$$U_{Spectator} = V_{ent} - (C_{ticket} + C_{abate)} \tag{5}$$

V_{ent} is the core entertainment value. Costs include the ticket price (C_{ticket}) and the cost of any emission abatement measures (C_{abate}), such as choosing lower-emission travel options or paying a carbon levy.

3. A repeated game with asymmetric information and reputation dynamics

To capture the complex, ongoing strategic interactions between the Tour (T) and its one Sponsor (S), this paper develops a model based on a repeated game with asymmetric information. This framework moves beyond static analysis to explore how short-term incentives are balanced against the long-term value of reputation, especially under the persistent threat of contract termination. The model is set in discrete time over an infinite horizon, t = 0, 1, 2, ...

3.1. Model setup

The Tour's Action: Continuous effort in each period t, the Tour chooses an abatement effort level $a_t \in [0, \infty)$, which is its private information. This effort incurs a convex cost $C(a_t)$(Equation (6)):

$$C_{abate,Tour} = C(a_t) = \frac{c}{2} a_t^2, \ where\ c > 0. \tag{6}$$

Signal and Imperfect Information: The Sponsor cannot directly observe at it. Instead, it observes a public, verifiable signal of environmental performance, x_t, which is a noisy measure of the Tour's true effort (Equation (7)):

$$x_t = a_t + \epsilon_t, \ where\ \epsilon_t \sim N(0, \sigma^2). \tag{7}$$

The variance σ^2 represents the degree of informational imperfection.

Reputation as a State Variable: The Sponsor's belief about the Tour's commitment is the state variable, which this paper calls reputation, μ_t. It is the Sponsor's prior belief at the start of period t. The Sponsor updates this belief via Bayes' Rule upon observing the signal x_t, leading to the posterior reputation for the next period, μ_{t+1}. For analytical tractability, this paper assume a linear updating rule: $\mu_{t+1} = (1-\gamma)\mu_t + \gamma x_t$, where is the learning rate or gain.

3.1.1. Payofs and decisions

The Sponsor (S): The Sponsor is assumed to be rational and makes its renewal decision in each period to maximize its own utility. The Sponsor's utility from the partnership in period t, $U_S(\mu_t)$, is the value derived from the association, $V(\mu_t)$, net of the sponsorship fee, S. This paper assumes is a strictly increasing function of the Tour's reputation, i.e., $V'(\mu_t) > 0$. If the Sponsor terminates the contract, it can invest in an outside option that yields a constant value V_{alt} .

The Sponsor's decision in period t is to solve the simple myopic optimization problem (Equation (8)):

$$max\{V(\mu_t) - S, V_{alt}\} \tag{8}$$

The Sponsor will choose to renew the contract if and only if the utility from doing so is at least as great as the utility from the outside option (Equation (9)):

$$V(\mu_t) - S \geq V_{alt} \tag{9}$$

This inequality implicitly defines the threshold strategy. This paper defines the minimum acceptable reputation level, as the level of reputation that makes the Sponsor exactly indifferent between renewing and terminating (Equation (10)):

$$V(\mu^*) - S = V_{alt} \Longrightarrow \mu^* = V^{-1}(V_{alt} + S) \tag{10}$$

Since V is a strictly increasing function, its inverse V^{-1} exists and is also strictly increasing. The Sponsor's optimal strategy is therefore a simple threshold rule:

• If $\mu_t \geq \mu^*$, then $V(\mu_t) \geq V(\mu^*)$, implying $V(\mu_t) - S \geq V_{alt}$. The Sponsor *renews*.

• If $\mu_t < \mu^*$, then $V(\mu_t) < V(\mu^*)$, implying $V(\mu_t) - S < V_{alt}$. The Sponsor *terminates*.

The Tour (T): The Tour's objective is to maximize its total discounted value stream. If the contract is renewed, it receives the sponsorship fee and incurs the cost $C(a_t)$. If the contract is terminated, its value stream reverts to a lower outside option value, V_0. The Tour's problem is therefore (Equation (11)):

$$\max_{\{a_t\}} \mathbb{E}\left[\sum_{t=0}^{\infty} \delta^t \pi_t\right] \tag{11}$$

where is the discount factor and is the per-period payoff, which depends on the Sponsor's decision derived above.

3.2. Equilibrium analysis: the dynamic optimization problem

The solution concept is Perfect Bayesian Equilibrium. The Tour's optimal effort strategy, $a^*(\mu)$, is derived from a Bellman equation that explicitly incorporates the Sponsor's termination rule.

3.2.1. The bellman equation with endogenous termination

The Tour's value function, $V_T(\mu)$, is a piece wise function of its reputation (Equation (12)):

$$V_{\mathrm{T}}(\mu) = \begin{cases} V_0 & \text{if } \mu < \mu^* \\ \max\limits_{a \geq 0} \{S - C(a) + \delta \mathbb{E}[V_{\mathrm{T}}(\mu') \mid \mu, a]\} & \text{if } \mu \geq \mu^* \end{cases} \tag{12}$$

Here, is the punishment payoff for failure to maintain reputation. The key insight is that the expectation term, $E[V_T(\mu')|\mu, a]$, must account for the possibility that the next-period reputation, μ', falls below the threshold μ^*.

3.2.2. The first-order condition of the bellman equation

The First-Order Condition (FOC) for the Tour's optimal effort, a^*, is found by maximizing the right-hand side of the Bellman equation for the case where the contract is renewed ($\mu \geq \mu^*$). The expression to be maximized with respect to the choice variable a is the sum of current-period net payoffs and the discounted expected future value (Equation (13)):

$$L(a) = S - C(a) + \delta E[V_T(\mu') \mid \mu, a] \tag{13}$$

To find the value of a that maximizes this expression, this paper takes the partial derivative of $L(a)$ with respect to a and set it equal to zero. This is the standard procedure for optimization. The FOC is found by setting the sum of the derivatives to zero (Equation (14)):

$$\tfrac{\partial L}{\partial a} = 0 - C'(a) + \delta \tfrac{\partial}{\partial a} E[V_T(\mu') \mid \mu, a] = 0 \tag{14}$$

This directly yields the condition (Equation (15)):

$$-C'(a) + \delta \tfrac{\partial}{\partial a} E[V_T(\mu')] = 0 \tag{15}$$

This condition embodies the economic principle that the Tour will choose an effort level a^* where the marginal cost of the last unit of effort, $C9(a^*)$, is exactly equal to the discounted marginal benefit of that effort, $\delta \tfrac{\partial}{\partial a} E[V_T(\mu')]$.

3.2.3. Calculating the marginal benefit term

Finding out $E[VT(\mu 9)]$ requires differentiating the expectation term with respect to effort a. First, this paper defines the critical threshold for the random shock ϵ. The contract is terminated if $\mu 9 < \mu^*$, which, using the linear updating rule, implies (Equation (16)):

$$(1-\gamma)\mu + \gamma(a+\epsilon) < \mu^* \Longrightarrow \epsilon < \tfrac{\mu^* - (1-\gamma)\mu}{\gamma} - a \tag{16}$$

Let us define this critical boundary as: Note that $\frac{d\epsilon^*(a)}{da} = -1$ The expectation can now be written in integral form, where $\phi(\epsilon)$ is the p.d.f. of the normal distribution (Equation (17)):

$$\mathbb{E}[V_T(\mu')] = \textstyle\int_{\epsilon^*(a)}^{\infty} V_T(\mu'(\mu, a, \epsilon))\phi(\epsilon)d\epsilon + \int_{-\infty}^{\epsilon^*(a)} V_0 \phi(\epsilon) d\epsilon \tag{17}$$

This paper differentiates this expression with respect to a using Leibniz's Integral Rule. Let's analyze each integral separately.

Derivative of the Continuation Value Integral (Equation (18)):

$$\begin{aligned} \tfrac{\partial}{\partial a} \textstyle\int_{\epsilon^*(a)}^{\infty} V_T(\mu')\phi(\epsilon)d\epsilon \;&= -V_T(\mu'(\mu, a, \epsilon^*(a)))\phi(\epsilon^*(a)) \cdot \tfrac{d\epsilon^*(a)}{da} + \textstyle\int_{\epsilon^*(a)}^{\infty} \tfrac{\partial}{\partial a}[V_T(\mu')]\phi(\epsilon)d\epsilon \\ &= -V_T(\mu^*)\phi(\epsilon^*(a)) \cdot (-1) + \textstyle\int_{\epsilon^*(a)}^{\infty} {V_T}'(\mu') \tfrac{\partial \mu'}{\partial a} \phi(\epsilon)d\epsilon \\ &= V_T(\mu^*)\phi(\epsilon^*(a)) + \gamma \textstyle\int_{\epsilon^*(a)}^{\infty} {V_T}'(\mu')\phi(\epsilon)d\epsilon / e \end{aligned} \tag{18}$$

Derivative of the Termination Value Integral (Equation (19)):

$$\begin{aligned} &\tfrac{\partial}{\partial a} \textstyle\int_{-\infty}^{\epsilon^*(a)} V_0 \phi(\epsilon) d\epsilon = V_0 \phi(\epsilon^*(a)) \cdot \tfrac{d\epsilon^*(a)}{da} \\ &= V_0 \phi(\epsilon^*(a)) \cdot (-1) = -V_0 \phi(\epsilon^*(a)) \end{aligned} \tag{19}$$

Combining these two results, the derivative of the full expectation term is (Equation (20)):

$$\begin{aligned}\frac{\partial}{\partial a}\mathbb{E}[V_T(\mu')] &= V_T(\mu^*)\phi(\epsilon^*(a)) + \gamma\int_{\epsilon^*(a)}^{\infty} V_T{}'(\mu')\phi(\epsilon)d\epsilon - V_0\phi(\epsilon^*(a)) \\ &= \gamma\int_{\epsilon^*(a)}^{\infty} V_T{}'(\mu')\phi(\epsilon)d\epsilon + [V_T(\mu^*) - V_0]\phi(\epsilon^*(a))\end{aligned} \tag{20}$$

3.2.4. The complete first-order condition and its interpretation

The first-order condition for the Tour's problem is $C,(a) + \delta\ \frac{\partial}{\partial a}\ E[V_T(\mu)] = 0$. Using $C,(a) = ca(Eq.14)$, the optimal effort a^* must satisfy (Equation (21)):

$$ca^* = \delta(\underbrace{\gamma\int_{\epsilon^*(a^*)}^{\infty} V_T{}'(\mu')\phi(\epsilon)d\epsilon}_{IntensiveMarginBenefit} + \underbrace{[V_T(\mu^*) - V_0]\phi(\epsilon^*(a^*))}_{ExtensiveMarginBenefit}) \tag{21}$$

This rigorously derived condition provides the following economic intuitions.

Marginal cost: The left side, ca^*, is the direct marginal cost of exerting effort.

Intensive margin benefit: The first term on the right is the marginal return from reputation-building. It is the expected marginal increase in future value, $E[V_T,(\mu)]$, conditional on the contract continuing, multiplied by the efficiency of effort in building reputation, γ. This is the benefit of making good outcomes even better.

Extensive margin benefit: The second term on the right is the marginal return from risk mitigation. It is the benefit of preventing contract termination. represents the capital value at stake, i.e., the total loss incurred if reputation falls just below the threshold, and $\phi(\epsilon^*(a^*))$ is the probability density at the critical termination boundary. It captures how a marginal increase in effort a shifts the boundary and reduces the probability mass in the termination region.

3.3. Properties and implications of the equilibrium

This complete model provides a robust foundation for several key implications.

Reputation as a capital asset: The model formalizes reputation as a tangible asset whose value is maintained through costly effort. The threat of losing this asset (i.e., the Sponsor terminating and the Tour's value dropping to V_0) provides the necessary discipline to deter shirking.

The economic value of transparency: Information transparency, represented by a low noise variance σ^2, is critical. Lower noise allows the Sponsor to more accurately infer effort from performance, which increases the density $\phi(\epsilon^*)$ near zero if the Tour is performing well, thus sharpening incentives. This provides a clear rationale for investing in high-quality, third-party verified carbon accounting.

The importance of long-term relationships: A high discount factor δ signifies that the Tour values the future highly. This increases the discounted value of both the intensive and extensive margin benefits of effort, making the Tour more willing to incur short-term costs for long-term reputational security. Stable, long-term partnerships are thus more conducive to credible sustainability strategies.

In conclusion, this dynamic model demonstrates that a Tour's optimal carbon compensation strategy is the result of a sophisticated optimization process. The Sponsor's termination threat is not an external constraint but is fully internalized into the Tour's decision-making, compelling it to invest in environmental effort as a means of risk management and long-term value preservation.

3.4. Numerical illustration of equilibrium properties

To provide a concrete illustration of the theoretical model's predictions, this paper solves the Tour's dynamic optimization problem numerically using Value Function Iteration on a discrete grid of the reputation state space, μ. By varying key parameters, this paper can visualize the shape of the optimal effort policy, $a^*(\mu)$, and confirm the qualitative properties derived from the first-order condition. The following experiments

demonstrate the impact of information transparency (σ^2) and the Tour's long-term orientation (δ) on its strategic effort choice. Relevant code used for the numerical solution is provided in Appendix A.

The economic value of transparency: First, this paper analyzes the effect of imperfect information, solve the model for three different levels of noise variance (σ^2), representing low, medium, and high degrees of transparency in monitoring the Tour's effort. A lower σ^2 implies that the public signal x_t is a more accurate measure of the true effort.

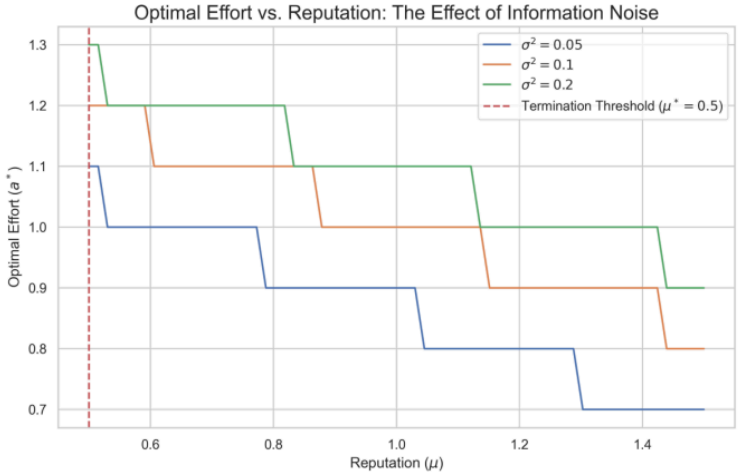


Figure 2. Optimal effort policy $a^*(\mu)$ for different levels of information noise (σ^2). The vertical dashed line indicates the sponsor's termination threshold $\mu^* = 0.5$

Figure 2 depicts the equilibrium. Specifically, a lower σ^2 makes the normal probability density function $\phi(\epsilon)$ more concentrated around zero. This magnifies the extensive margin benefit of effort, $[V_T(\mu^*) - V_0]\phi(\epsilon^*(a^*))$, as a marginal increase in effort has a larger impact on reducing the probability of falling into the termination region.

The Importance of Long-Term Relationships. A higher δ signifies that the Tour places greater weight on future payoffs relative to present costs.

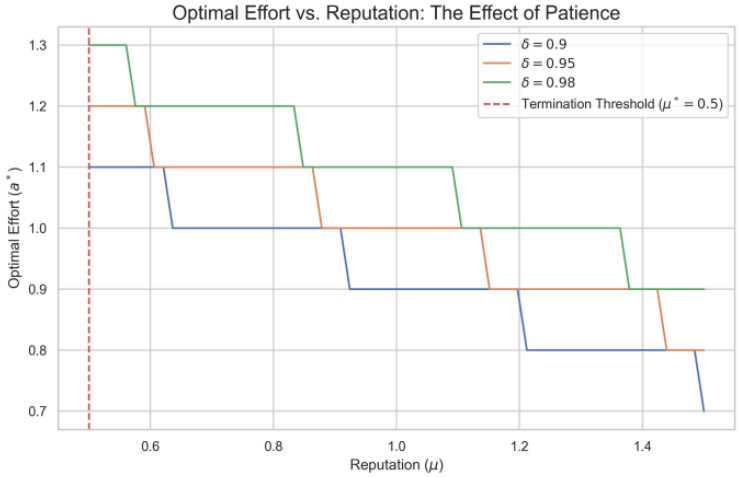


Figure 3. Optimal effort policy $a^*(\sigma)$ for different discount factors (δ). A higher δ reflects greater patience

As shown in Figure 3, a higher discount factor results in a significantly higher level of optimal effort across the entire reputation space. This finding directly supports the theoretical model. The discount factor δ scales the entire future value component in the Bellman equation. Consequently, a more patient Tour perceives a greater long-term return from maintaining its reputation, making it more willing to incur the short-term costs of abatement effort. This highlights that stable, long-term partnerships are more conducive to credible and sustained pro-environmental strategies, as the long shadow of the future incentivizes present-day investment.

In both scenarios, it is also visually evident that the optimal effort $a^*(\mu)$ increases sharply as reputation μ approaches the threshold μ^*. This nonlinear response underscores the role of reputation as a capital asset and the critical importance of the extensive margin benefit in compelling the Tour to actively manage the risk of termination.

4. Optimal abatement mechanism under asymmetric information

4.1. The principal-agent framework

This paper models the strategic interaction between the Tour (the Principal) and the individual Tournaments (the Agents) using a Bayesian principal-agent framework. The Tour's objective is to minimize the total social cost of carbon emissions, which comprises the aggregate cost of abatement activities and the cost of damage from residual emissions. The central challenge is that the Tour does not know the true abatement cost of each individual Tournament; this information is private to the Tournament itself.

Let there be a continuum of Tournaments, indexed by their type $\theta \in [\underline{\theta}, \bar{\theta}]$. The type θ is a privately known parameter that determines the Tournament's efficiency in emissions abatement. A higher θ signifies a higher marginal cost of abatement. The Tour only knows the probability distribution of types, characterized by a probability density function (p.d.f.) and a cumulative distribution function (c.d.f.) $F(\theta)$, with and $F(\bar{\theta}) = 1$.

Let be the baseline emissions level for any Tournament in the absence of regulation. An abatement level results in an emissions level $E = \overline{E} - a$. The cost of abatement for a Tournament of type is given by a convex function $C(a, \theta)$. For analytical tractability, this paper assumes a quadratic form (Equation (22)):

$$C(a, \theta) = \frac{\theta}{2} a^2 = \frac{\theta}{2} (\overline{E} - E)^2 \tag{22}$$

The environmental damage from total emissions is given by a convex function $D(E_{total})$, where $E_{total} = \int_{\underline{\theta}}^{\bar{\theta}} E(\theta) f(\theta) d\theta$. The Tour's problem is to design a mechanism that specifies an emissions target $E(\theta)$ and a monetary transfer $T(\theta)$ for each possible type θ. The transfer $T(\theta)$ can be positive (a subsidy) or negative (a fee, equivalent to fee in the main model). A Tournament of type θ that is assigned the pair (E, T) receives a net utility (or profit) of (Equation (23)):

$$\pi(\theta; E, T) = -T - C(\overline{E} - E, \theta) = -T - \frac{\theta}{2} (\overline{E} - E)^2 \tag{23}$$

4.2. The direct revelation mechanism

By the Revelation Principle, this paper can restrict the attention to direct mechanisms where the Tour asks each Tournament to report its type, $\hat{\theta}$, and designs the mechanism such that all Tournaments find it optimal to report their true type, i.e., $\hat{\theta} = \theta$. Such a mechanism must satisfy two key constraints:

Incentive Compatibility (IC): Each Tournament must receive a higher utility from truthfully reporting its type than from misrepresenting it. For any true type and any reported type $\hat{\theta}$ (Equation (24)):

$$-T(\theta) - \frac{\theta}{2} (\overline{E} - E(\theta))^2 \geq -T(\hat{\theta}) - \frac{\theta}{2} (\overline{E} - E(\hat{\theta}))^2, \ \forall \theta, \hat{\theta} \in [\underline{\theta}, \bar{\theta}] \tag{24}$$

Individual Rationality (IR): Each Tournament must achieve a non-negative utility from participating in the mechanism. The reservation utility is normalized to zero (Equation (25)).

$$-T(\theta)-\frac{\theta}{2}(\overline{E}-E(\theta))^2\geq 0,\ \forall\theta\in[\underline{\theta},\bar{\theta}] \tag{25}$$

4.3. Characterization of incentive compatibility

Let be the equilibrium utility of a Tournament of type that reports truthfully (Equation (26)):

$$U(\theta)=-T(\theta)-\frac{\theta}{2}(\overline{E}-E(\theta))^2 \tag{26}$$

The IC constraint implies that $U(\theta)=max_{\hat{\theta}}\{-T(\hat{\theta})-\frac{\theta}{2}(\overline{E}-E(\hat{\theta}))^2\}$. By the Envelope Theorem, this paper can differentiate with respect to θ (Equation (27)):

$$\begin{aligned}\frac{dU(\theta)}{d\theta}&=\frac{\partial}{\partial\theta}[-T(\hat{\theta})-\frac{\theta}{2}(\overline{E}-E(\hat{\theta}))^2]|_{\hat{\theta}=\theta}\\ \Longrightarrow\frac{dU(\theta)}{d\theta}&=-\frac{1}{2}(\overline{E}-E(\theta))^2\qquad(*)\end{aligned} \tag{27}$$

This differential equation is a necessary condition for incentive compatibility. It shows that the utility of a Tournament decreases as its marginal cost parameter increases, and the rate of decrease is faster for Tournaments assigned a larger abatement task (a smaller $E(\theta)$).

To find the level of utility, this paper integrates this equation. The IR constraint, $U(\theta)\geq 0$, must hold for all θ. Since is a decreasing function of θ, the IR constraint is most binding for the least efficient type, $\bar{\theta}$. To minimize the payments to the Agents, the Principal will leave the least efficient type with zero utility: $U(\bar{\theta})=0$. Integrating from to $\bar{\theta}$ (Equations (28) and (29)):

$$\int_{\theta}^{\bar{\theta}}\frac{dU(s)}{ds}ds=\int_{\theta}^{\bar{\theta}}-\frac{1}{2}(\overline{E}-E(s))^2ds \tag{28}$$

$$U(\bar{\theta})-U(\theta)=-\int_{\theta}^{\bar{\theta}}\frac{1}{2}(\overline{E}-E(s))^2ds \tag{29}$$

Substituting $U(\bar{\theta})=0$, this paper solves for $U(\theta)$ (Equation (30)):

$$U(\theta)=\int_{\theta}^{\bar{\theta}}\frac{1}{2}(\overline{E}-E(s))^2ds \tag{30}$$

This is the information rent captured by a Tournament of type θ. It is the extra utility a Tournament earns due to its private information. To induce truth-telling, the Tour must allow more efficient types (lower θ) to capture this rent.

From the definition of $U(\theta)$, this paper can express the transfer as (Equation (31)):

$$T(\theta)=-U(\theta)-\frac{\theta}{2}(\overline{E}-E(\theta))^2 \tag{31}$$

4.4. The principal's optimization problem

The Tour's objective is to choose the allocation function $E(\theta)$ to maximize total surplus minus the expected information rent paid to the agents. This is equivalent to minimizing the sum of expected abatement costs and expected information rents, plus the total environmental damage. The objective function is (Equation (32)):

$$\min_{\mathbf{E}(\theta)}\int_{\underline{\theta}}^{\bar{\theta}}[C(\mathbf{E}(\theta),\theta)+U(\theta)]f(\theta)\mathrm{d}\theta+D\left(\int_{\underline{\theta}}^{\bar{\theta}}\mathbf{E}(\theta)f(\theta)\mathrm{d}\theta\right) \tag{32}$$

Substituting the integral form of $U(\theta)$ (Equation (33)):

$$\min_{\mathbf{E}(\theta)}\int_{\underline{\theta}}^{\bar{\theta}}\left[\frac{\theta}{2}(\overline{\mathbf{E}}-\mathbf{E}(\theta))^2+\int_{\underline{\theta}}^{\theta}\frac{1}{2}(\overline{\mathbf{E}}-\mathbf{E}(s))^2\mathrm{d}s\right]f(\theta)\mathrm{d}\theta+D(\cdot) \tag{33}$$

This paper simplifies the expected information rent term using integration by parts on $\int U(\theta)f(\theta)d\theta$. Let and $v = U(\theta)$. Then and $v' = U'(\theta) = -\frac{1}{2}\left(\overline{E} - E(\theta)\right)^2$ (Equation (34)).

$$\begin{aligned} &\int_{\underline{\theta}}^{\bar{\theta}} U(\theta)f(\theta)d\theta = [U(\theta)F(\theta)]_{\underline{\theta}}^{\bar{\theta}} - \int_{\underline{\theta}}^{\bar{\theta}} U'(\theta)F(\theta)d\theta \\ &= (U(\bar{\theta})F(\bar{\theta}) - U(\underline{\theta})F(\underline{\theta})) - \int_{\underline{\theta}}^{\bar{\theta}} (-\tfrac{1}{2}(\overline{E} - E(\theta))^2)F(\theta)d\theta \\ &= (0 \cdot 1 - U(\underline{\theta}) \cdot 0) + \int_{\underline{\theta}}^{\bar{\theta}} \tfrac{F(\theta)}{2}(\overline{E} - E(\theta))^2 d\theta \\ &= \int_{\underline{\theta}}^{\bar{\theta}} \tfrac{F(\theta)}{f(\theta)} \tfrac{1}{2}(\overline{E} - E(\theta))^2 f(\theta)d\theta \end{aligned} \tag{34}$$

Substituting this back into the Principal's objective function, this paper gets (Equation (35)):

$$\begin{aligned} &\min_{\mathbf{E}(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \left[\tfrac{\theta}{2}\left(\overline{\mathbf{E}} - \mathbf{E}(\theta)\right)^2 + \tfrac{f(\theta)}{f(\theta)} \tfrac{1}{2}\left(\overline{\mathbf{E}} - \mathbf{E}(\theta)\right)^2\right] f(\theta)\mathrm{d}\theta + D(\cdot) \\ &\Rightarrow \min_{\mathbf{E}(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \left[\theta + \tfrac{f(\theta)}{f(\theta)}\right] \tfrac{1}{2}\left(\overline{\mathbf{E}} - \mathbf{E}(\theta)\right)^2 f(\theta)\mathrm{d}\theta + D\left(\int_{\underline{\theta}}^{\bar{\theta}} \mathbf{E}(\theta)f(\theta)\mathrm{d}\theta\right) \end{aligned} \tag{35}$$

4.5. The optimal allocation rule

The objective is now a standard calculus of variations problem. This paper can solve it by performing a pointwise minimization for each θ . The first-order condition with respect to is (Equation (36)):

$$-[\theta + \tfrac{F(\theta)}{f(\theta)}](\overline{E} - E^*(\theta))f(\theta) + D'(\int_{\underline{\theta}}^{\bar{\theta}} E^*(s)f(s)ds)f(\theta) = 0 \tag{36}$$

Dividing by and rearranging yields the optimal allocation rule (Equation (37)):

$$\underbrace{\theta(\overline{E} - E^*(\theta))}_{MarginalAbatementCost} + \underbrace{\tfrac{F(\theta)}{f(\theta)}\left(\overline{E} - E^*(\theta)\right)}_{MarginalInformationRentCost} = \underbrace{D'(E[E^*(\theta)])}_{MarginalSocialDamage} \tag{37}$$

The term on the left is the virtual marginal cost. It consists of the true marginal abatement cost for type plus a non-negative markup. This markup represents the marginal cost of the information rent that must be paid to all more efficient types (i.e., types $s < \theta$) if type is asked to abate more.

This rule has two important properties:

•No Distortion at the Bottom: For the most efficient type, $\underline{\theta}$, this paper has $F(\underline{\theta}) = 0$. The markup is zero, and its marginal abatement cost is set equal to the marginal social damage. The allocation is first-best efficient.

•Distortion for all other types: For all $\theta > \underline{\theta}$, the virtual cost is strictly greater than the true marginal cost. This implies that $(\overline{E} - E^*(\theta))$, the abatement level, will be lower than the first-best level. To reduce the information rents paid to efficient types, the Principal optimally distorts the allocation for less efficient types by assigning them less ambitious abatement targets.

The optimal fee schedule, $T^*(\theta)$, is then found by substituting the optimal allocation back into the equations for and $T(\theta)$. The resulting fee is necessarily non-linear and type-dependent to ensure incentive compatibility.

4.6. Analytical solution under specific assumptions

To move from the general characterization to a concrete solution, this paper analyzes the model under a set of standard, tractable assumptions. This paper assumes the Tournament types are uniformly distributed and the marginal environmental damage is constant. Specifically:

•Uniform Distribution of Types: The abatement cost parameter is uniformly distributed over the interval $[\underline{\theta}, \bar{\theta}]$. This implies a probability density function (p.d.f.) and a cumulative distribution function (c.d.f.)

$F(\theta) = \frac{\theta-\underline{\theta}}{\bar{\theta}-\underline{\theta}}$. A key implication is that the inverse hazard rate term becomes linear in θ (Equation (38)):

$$\frac{F(\theta)}{f(\theta)} = \frac{(\theta-\underline{\theta})/(\bar{\theta}-\underline{\theta})}{1/(\bar{\theta}-\underline{\theta})} = \theta - \underline{\theta} \tag{38}$$

•Linear Marginal Damage: This paper assumes the marginal social damage from emissions is a constant, λ. This is equivalent to assuming the Tour has a fixed carbon price or target, such that $D'(\mathbb{E}[E^*(\theta)]) = \lambda$. This simplifies the analysis by removing the feedback loop where the allocation itself determines the marginal damage.

4.6.1. Derivation of the optimal abatement and fee schedule

Substituting these assumptions into the optimal allocation rule from Equation (20), this paper has (Equation (39)):

$$\begin{aligned}[\theta + (\theta - \underline{\theta})](\overline{E} - E^*(\theta)) = \lambda \\ (2\theta - \underline{\theta})(\overline{E} - E^*(\theta)) = \lambda\end{aligned} \tag{39}$$

This equation can now be solved directly for the optimal abatement level, $a^*(\theta) = \overline{E} - E^*(\theta)$ (Equation (40)):

$$a^*(\theta) = \frac{\lambda}{2\theta-\underline{\theta}} \tag{40}$$

And consequently, the optimal emissions target for a Tournament of type is (Equation (41)):

$$E^*(\theta) = \overline{E} - \frac{\lambda}{2\theta-\underline{\theta}} \tag{41}$$

This solution demonstrates that the required abatement is a decreasing function of the Tournament's cost parameter θ, which is a necessary condition for incentive compatibility. More efficient Tournaments (lower θ) are required to abate more.

With the optimal allocation rule defined, this paper can now derive the information rent and the transfer $T(\theta)$. The information rent is (Equation (42)):

$$\begin{aligned}U(\theta) = \int_{\theta}^{\bar{\theta}} \frac{1}{2}(\overline{E} - E(s))^2 ds = \int_{\theta}^{\bar{\theta}} \frac{1}{2}(a^*(s))^2 ds \\ = \int_{\theta}^{\bar{\theta}} \frac{1}{2}\left(\frac{\lambda}{2s-\underline{\theta}}\right)^2 ds = \frac{\lambda^2}{2}\int_{\theta}^{\bar{\theta}}(2s-\underline{\theta})^{-2} ds \\ = \frac{\lambda^2}{2}\left[-\frac{1}{2(2s-\underline{\theta})}\right]_{\theta}^{\bar{\theta}} \\ = \frac{\lambda^2}{4}\left(\frac{1}{2\theta-\underline{\theta}} - \frac{1}{2\bar{\theta}-\underline{\theta}}\right)\end{aligned} \tag{42}$$

Finally, this paper can determine the transfer (fee/subsidy) required to implement this outcome (Equation (43)):

$$\begin{aligned}T^*(\theta) = -U(\theta) - \frac{\theta}{2}(a^*(\theta))^2 \\ = -\frac{\lambda^2}{4}\left(\frac{1}{2\theta-\underline{\theta}} - \frac{1}{2\bar{\theta}-\underline{\theta}}\right) - \frac{\theta}{2}\left(\frac{\lambda}{2\theta-\underline{\theta}}\right)^2\end{aligned} \tag{43}$$

This expression defines the non-linear fee schedule. The fee paid by a tournament depends critically on its type θ (which it reveals through its choice of contract), and is composed of two parts: a clawback of the information rent and the reimbursement of its (distorted) abatement costs.

4.6.2. Managerial and policy implications

The analytical solution provides several crucial insights for the Tour's design of a carbon compensation mechanism here.

The Inefficiency of Uniform Regulation: The derived optimal fee $T^*(\theta)$ and abatement target $a^*(\theta)$ are both highly non-linear functions of the Tournament's type θ. This proves that a simple, one-size-fits-all regulation, such as a uniform carbon tax or a fixed emission cap for all events, would be suboptimal. To be efficient in the face of private information, the mechanism must offer a menu of options that allows for self-selection, effectively discriminating between high-cost and low-cost abaters.

Efficiency for the Best, Distortion for the Rest: The model confirms the principle of "no distortion at the bottom". For the most efficient Tournament type, $\theta = \underline{\theta}$, the allocation rule becomes $\underline{\theta} \cdot a^*(\underline{\theta}) = \lambda$. This is the first-best outcome, where the marginal cost of abatement equals the marginal social damage. There is no incentive to distort this type's behavior because there are no more-efficient types to pay information rents to. For all other types ($\theta > \underline{\theta}$), the virtual marginal cost is strictly greater than the true marginal cost θ. This leads to an abatement level that is strictly lower than the first-best level.

The Economic Rationale for Under-Regulation: The downward distortion for less efficient types is not a design flaw but an optimal response to asymmetric information. To persuade the most efficient tournaments (low θ) to undertake significant abatement, they must be rewarded with an information rent. To minimize this rent payout, the Tour deliberately makes the contracts designed for less efficient tournaments (high θ) less attractive by assigning them lower abatement targets. This calculated 'under-regulation' of inefficient agents is the price the Principal must pay to manage the information asymmetry problem effectively.

Information Rent as a Necessary Incentive: The term quantifies the extra utility (i.e., profit) that a Tournament with cost must be allowed to keep to voluntarily participate and truthfully reveal its type. As shown, is decreasing in θ, meaning more efficient Tournaments capture higher rents. This insight is critical for budgeting: a portion of the funds in any carbon compensation scheme must be explicitly understood not as a cost reimbursement, but as a necessary incentive payment to overcome the information gap. Any attempt to confiscate this rent would cause the mechanism to unravel as efficient agents would have an incentive to misrepresent their costs.

In conclusion, this analytical framework demonstrates that the optimal mechanism is not a simple tax but a sophisticated screening device. It strategically balances the direct costs of abatement against the indirect costs of information rents, providing a clear theoretical foundation for designing incentive-compatible environmental regulations within a complex multi-agent system like a professional sports tour.

4.7. Computational experiments: abatement schedules and welfare comparisons

This subsection augments the theoretical analysis with simulation evidence. The goal is twofold. First, this paper visualizes how abatement requirements vary across tournaments with different abatement cost parameters θ, under alternative policy instruments. Second, this paper compares the implied welfare outcomes to highlight the efficiency gains delivered by the incentive-compatible (IC/IR) mechanism relative to a one-size-fits-all benchmark.

Modeling background. Recall that there is a continuum of tournaments indexed by type $\theta \in [\underline{\theta}, \bar{\theta}]$, where higher indicates a higher marginal cost of abatement. The baseline (unregulated) emissions level is per tournament, and abatement yields emissions $E = \overline{E} - a$. Abatement costs are quadratic, $C(a, \theta) = \frac{\theta}{2} a^2$, capturing convex disutility from deeper cuts. Social damages depend on total emissions, and in the simulations this paper adopts a linear marginal damage specification $D'(E_{total}) = \lambda$, consistent with the analytical section where this paper takes as an exogenous marginal social damage (e.g., a shadow carbon price). Types are drawn from a uniform distribution, $\theta \sim Uniform[\underline{\theta}, \bar{\theta}]$.

Mechanisms compared. This paper implements three allocation rules that map types into abatement levels a(θ), capping abatement at E for feasibility:

(1) First-best (full information): With perfect information, the planner equalizes marginal abatement cost and marginal damage pointwise: $\theta \cdot a^{FB}(\theta) = \lambda$, so $a^{FB}(\theta) = \lambda/\theta$. This provides an efficiency lower bound on (real-resource) social cost.

(2) Second-best (IC/IR): Under asymmetric information, the planner faces an informat5i6o3n- rent externality. The optimal direct mechanism solves the virtual-cost condition (Equation (44)).

$$\left(\underbrace{\theta}_{true\ cost} + \underbrace{F(\theta)/f(\theta)}_{true\ rentmarkup} \right) a^{SB}(\theta) = \lambda \tag{44}$$

Under a uniform distribution this reduces to $(2\theta - \underline{\theta}) \cdot a^{SB}(\theta) = \lambda$, hence (Equation (45)):

$$a^{SB}(\theta) = \frac{\lambda}{2\theta - \underline{\theta}} \tag{45}$$

This rule preserves no distortion at the bottom ($\theta = \underline{\theta}$) and prescribes optimally *downward-distorted* abatement relative to the first-best for all $\theta > \underline{\theta}$, thereby economizing on information rents.

(3) Uniform tax (naive benchmark): A one-size-fits-all carbon price T induces private abatement aUT (θ) = T/θ. Because it ignores heterogeneity, it generally fails to achieve the screening benefits of the second-best mechanism.

Figure 4 shows the simulated schedules a(θ) in levels. It demonstrates theoretical signatures: (i) No distortion at the bottom: the Second-best and First-best coincide at θ because no more efficient types exist to whom rents would be paid. (ii) Downward distortion otherwise: for all $\theta > \underline{\theta}$, Second-best requires less abatement than First-best, optimally trading off environmental gains against the marginal cost of information rents. By contrast, a uniform carbon price $aUT(\theta) = T/\theta$ misses this trade-off and does not implement the virtual-cost markup, leading to systematic misallocation across types.

Efficiency advantage over uniform policies. This paper also analysed how total social cost changes with marginal damage. As shown in Figure 5, across a wide range of λ, the Second-best achieves lower total social cost than the uniform tax. Intuitively, the virtual- cost markup $\theta + \frac{F(\theta)}{f(\theta)}$ internalizes the shadow cost of information rents and rebalances

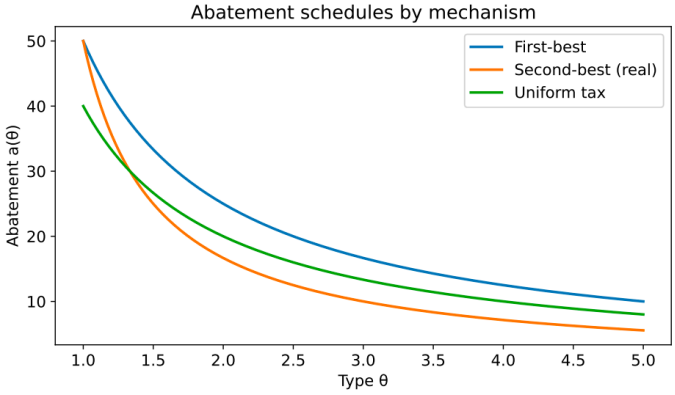


Figure 4. Abatement schedules by mechanism. Each curve is decreasing in θ, reflecting higher marginal abatement costs for less efficient tournaments

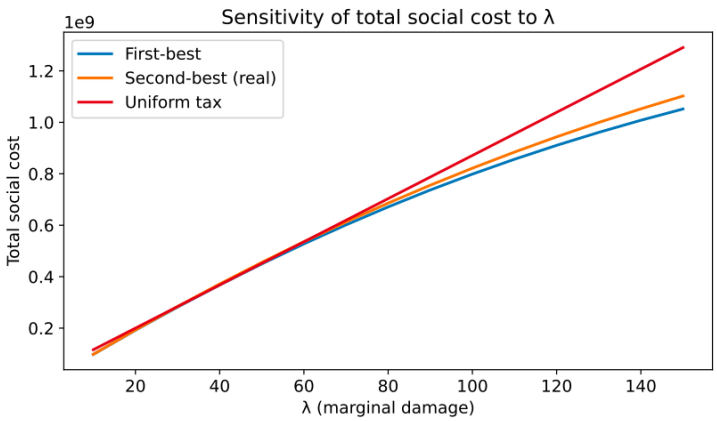


Figure 5. Sensitivity of total social cost to λ (real-resource metric)

Sensitivity analysis shows that while higher λ raises the social value of abatement, the ranking of mechanisms remains stable: First-best is a theoretical lower bound; Second-best remains strictly closer to that bound than a uniform tax for generic calibrations. Hence, in environments with persistent private information, a menu-based, IC/IR mechanism should be preferred to uniform instruments on efficiency grounds.

Sensitivity analysis shows that while higher λ raises the social value of abatement (and thus total abatement efforts), the ranking of mechanisms remains stable: First-best is a theoretical lower bound; Second-best remains strictly closer to that bound than a uniform tax for generic calibrations. Hence, in environments with persistent private information, a menu-based, IC/IR mechanism should be preferred to uniform instruments on efficiency grounds.

5. An integrated dynamic mechanism design model

Section 3 and Section 4 analyzed the Tour's strategic challenges in two distinct stages: managing long-term reputation with its Sponsor and designing a static regulatory mechanism for its Tournaments. A limitation of that approach is the exogenous nature of the environmental objective in the mechanism design problem, represented by the marginal damage function $D,(\cdot)$. In reality, the Tour's commitment to environmental regulation is not completely driven by an abstract social good, but by the same private incentive that drives its relationship with the Sponsor: the preservation of its reputational and commercial capital.

Based on the previous 2 models, this section develops an integrated model that unifies these two problems. This paper constructs a dynamic mechanism design framework where the Tour, in each period, chooses an entire regulatory mechanism as its strategic action to manage its long-term reputation. This endogenizes the environmental target, revealing how the stringency of regulation should dynamically respond to the Tour's reputational standing.

5.1. Model setup

This paper maintains the infinite-horizon, discrete-time framework. The state variable is the Tour's public reputation, μ_t:

Algorithm 1 logic for a single game period (t) is as follows:

1: Input: Current reputation μ_t, Model parameters $(\mu^*, V_0, \gamma, \sigma^2)$

2: Output: Next period's reputation μ_{t+1} or game termination signal

3: procedure RunPeriod(μ_t)

▷*Step1:The periodbegins with a known reputation level.*

4: Let current_reputation ← μ_t

▷*Step 2:The Sponsor's renewal decision.*

5: if current_reputation < μ^* then

6: Tour_Payoff ← V_0

7: return TERMINATE GAME

8: else ▷*Contract isrenewed for the period.*

▷*Step3:TheTour(Principal)designstheoptimal mechanism.*

9: M_t← Design Optimal Mechanism(μ_t) ▷Returns the functions $\{E_t(\theta), T_t(\theta)\}$

▷*Step 4 & 5:Agents interact, and the mechanism yields an outcome.*

10: A_t← Execute Mechanism*(M_t)* ▷Calculates aggregate abatement

▷*Step6:TheSponsor observes a noisy public signal.*

11: ϵ_t← $N(0, \sigma^2)$ ▷ Draw a noise term from a normal distribution

12:x_t← A_t + ϵ_t

▷*Step7:Beliefs are updated todetermine the next period's state.*

13: $\mu_{t+1} \leftarrow (1-\gamma)\cdot\mu_t + \gamma\cdot x_t$

14: return μ_{t+1}

15: end if

16: end procedure

Algorithm 2 Timing of the Game — The sequence of actions within a single period t is as follows (summarize this in Algorithm 1):

1. The period begins with a known reputation level μ_t.

2. The Sponsor observes μ_t. If $\mu_t < \mu^*$, the sponsorship contract is terminated, and the Tour receives a perpetual punishment value of V_0. If $\mu_t \geq \mu^*$, the contract is renewed for the period.

3. If renewed, the Tour (Principal) designs and commits to a direct revelation mechanism, $M_t = E_t(\theta), T_t(\theta)_{\theta\in[\underline{\theta},\bar{\theta}]}$, which is offered to the continuum of Tournaments (Agents).

4. Tournaments report their privately known cost types θ, and the mechanism is executed.

5. The mechanism produces an aggregate level of abatement for the period, A_t.

6. The Sponsor does not observe A_t directly but sees a noisy public signal x_t = A_t + ϵ_t, where ϵ_t ~ $N(0, \sigma^2)$.

7. Beliefs are updated via the linear rule, $\mu_{t+1} = (1-\gamma)\mu_t + \gamma x_t$, and the game proceeds to the next period.

5.2. The dynamic programming problem

The Tour's objective is to choose a sequence of regulatory mechanisms $\{E_t(\theta)\}_{t=0}^{\infty}$ to maximize its total discounted value. Its value function, $V_T(\mu)$, must satisfy the following Bellman equation (Equation (46)):

$$V_T(\mu) = \begin{cases} V_0 & if \mu < \mu^* \\ \max\limits_{E(\theta)} \{S - C_{mech}(E(\theta)) + \delta \mathbb{E}[V_T(\mu') \mid \mu, A[E(\theta)]]\} & if \mu \geq \mu^* \end{cases} \tag{46}$$

where the choice variable is the entire allocation function $E(\theta)$.

S is the fixed sponsorship fee received in the period. Cmech ($E(\theta)$) is the total cost of implementing the incentive-compatible mechanism. As derived in Section 4, this is the expected virtual cost, which includes both the real abatement costs incurred by Tournaments and the information rents paid to them, $C_{mech}(E(\theta)) = \int_{\underline{\theta}}^{\bar{\theta}} [\theta + \frac{F(\theta)}{f(\theta)}] \frac{1}{2} (\overline{E} - E(\theta))^2 f(\theta) d\theta \cdot \delta E[V_T(\mu)|...]$ is the discounted expected continuation value, which depends on how the chosen mechanism $E(\theta)$ affects the aggregate abatement A and, consequently, the evolution of reputation μ.

5.3. The optimal dynamic allocation rule

To find the optimal mechanism $E^*(\theta)$, this paper solves the Tour's maximization problem. This is a problem in the calculus of variations, where the first-order condition must hold for each type $\theta \in [\underline{\theta}, \bar{\theta}]$. The condition balances the marginal virtual cost of requiring more abatement from type θ against the marginal future reputational benefit generated by the resulting increase in aggregate abatement A.

Following a derivation that combines the logic of Section 3 and Appendix B, this study arrives at the optimal dynamic allocation rule. The abatement level $a^*(\theta) = \underline{E} - E^*(\theta)$ for each type θ must satisfy (Equation (47)):

$$\underbrace{(\theta + \frac{F(\theta)}{f(\theta)}) a^*(\theta)}_{\times} = \underbrace{\delta(\gamma \int_{\epsilon^*(A^*)}^{\infty} V_T\prime(\mu\prime) \phi(\epsilon) d\epsilon + [V_T(\mu^*) - V_0] \phi(\epsilon^*(A^*)))}_{\times} \tag{47}$$

where A^* is the aggregate abatement generated by the optimal mechanism, $A^* = \int a^*(\theta) f(\theta) d\theta$.

5.4. Interpretation of the equilibrium condition

Equation (47) provides a powerful synthesis of the entire framework and is the central normative result of the integrated model.

The left-hand side: the logic of regulation—The LHS is identical to the virtual marginal cost from the static analysis in Section 4. It represents the full marginal cost to the Tour of demanding an additional unit of abatement from a Tournament of type θ. This cost includes not only the real resource cost ($\theta \cdot a^*(\theta)$) but also the marginal information rent that must be ceded to all more efficient types ($\frac{F(\theta)}{f(\theta)} a^*(\theta)$).

The right-hand side: the logic of reputation—The RHS is identical in structure to the marginal benefit of effort from this paper' s reputational model in Section 3. It represents the shadow price of reputation, i.e., the total marginal value the Tour gains from a one- unit increase in its aggregate abatement A. This value comprises (1) An intensive margin benefit: The expected increase in continuation value from improving the Tour's reputation, conditional on the contract being renewed; and (2) An extensive marginal benefit: The value of reducing the probability of termination by moving further away from the reputational threshold μ^*.

Key insight: endogenous and state-dependent regulation—The crucial insight is that the environmental target is no longer an exogenous parameter λ but is now a endogenous shadow price that depends on the Tour' s reputational state μ. The RHS o Equation (47) is not a constant; its value changes as μ changes. This implies that the Tour' s optimal environmental policy is inherently dynamic and state-contingent:

•When reputation is secure (i.e., μ is far above μ^*), the risk of termination i low. The extensive margin benefit on the RHS is negligible. The Tour can afford to implement a less stringent regulatory mechanism (a lower $a^*(\theta)$ for all types) focusing on minimizing current-period mechanism costs.

•When reputation is precarious (i.e., μ is close to μ^*), the threat of termination is acute. The extensive margin benefit becomes very large, dramatically increasing the RHS. The Tour is compelled to implement a highly stringent mechanism (a higher $a^*(\theta)$ for all types), effectively signaling its commitment to reform, even at a high short-term cost.

In conclusion, the integrated model reveals that a sophisticated organization should not adopt a static environmental policy. Instead, it should treat its regulatory stringency as a flexible instrument for managing its most critical intangible asset: its reputation.

6. Internal mechanism design: the tournament's delegation problem

6.1. The cost minimization framework

Once a specific Tournament has been assigned an emissions target by the Tour's higher-level mechanism, its strategic problem changes. The Tournament now acts as a local Principal facing its own set of agents: the participating players ($j = 1, \ldots, n_i$) and spectators ($l = 1, \ldots, m_i$). The problem is no longer one of eliciting private information about its own aggregate abatement costs, but rather one of efficiently delegating its emissions target to its constituents to meet the goal at the minimum possible internal cost.

The Tournament's objective is to solve the following constrained optimization problem (Equation (48)):

$$\min_{\{e_{\mathrm{P},j}, e_{\mathrm{S},l}\}} \left(\sum_{j=1}^{n_i} C_{\mathrm{P},j}(e_{\mathrm{P},j}) + \sum_{l=1}^{m_i} C_{\mathrm{S},l}(e_{\mathrm{S},l}) \right) \tag{48}$$

subject to the constraint that its total emissions do not exceed the mandated target (Equation (49)):

$$\sum_{j=1}^{n_i} e_{P,j} + \sum_{l=1}^{m_i} e_{S,l} \leq E_i^* \tag{49}$$

Here, and are the individual emissions of player and spectator l , respectively. The functions and represent their individual abatement cost functions, which are assumed to be convex. For a given individual with baseline emissions $\bar{e}$, their abatement is $a = \bar{e} - e$, and the cost is a function of this abatement.

6.2. The lagrangian method and the equimarginal principle

This is a convex optimization problem that can be solved using the Lagrangian method. This paper formulates the Lagrangian, L, where λ is the Lagrange multiplier associated with the emissions constraint. This multiplier can be interpreted as the shadow price of emissions for the Tournament (Equation (50)).

$$L = \sum_{j=1}^{n_i} C_{P,j}(e_{P,j}) + \sum_{l=1}^{m_i} C_{S,l}(e_{S,l}) + \lambda(\sum_{j=1}^{n_i} e_{P,j} + \sum_{l=1}^{m_i} e_{S,l} - E_i^*) \tag{50}$$

The first-order necessary conditions for a minimum require that the partial derivatives of with respect to each individual's emissions level be equal to zero (Equation (51)):

$$\begin{aligned} \frac{\partial L}{\partial e_{P,j}} &= \frac{dC_{P,j}}{de_{P,j}} + \lambda = 0 \quad for\ all\ j = 1, \ldots, n_i \\ \frac{\partial L}{\partial e_{S,l}} &= \frac{dC_{S,l}}{de_{S,l}} + \lambda = 0 \quad for\ all\ l = 1, \ldots, m_i \end{aligned} \tag{51}$$

The derivative of the cost function with respect to emissions, $\frac{dC}{de}$, is the negative of the Marginal Abatement Cost (MAC), since $a = \bar{e} - e \Longrightarrow da = -de$. Thus, $\frac{dC}{de} = \frac{dC}{da}\frac{da}{de} = -MAC$. Substituting this into the first-order conditions, this paper obtain the central result (Equation (52)):

$$MAC_{P,j}(e^*_{P,j}) = \lambda \quad and \quad MAC_{S,l}(e^*_{S,l}) = \lambda \tag{52}$$

This is the equimarginal principle of cost minimization. It implies that to achieve a given total emissions reduction at the lowest possible aggregate cost, the marginal cost of abatement must be equalized across all contributing agents. If one agent had a lower MAC than another, total cost could be reduced by reallocating abatement effort from the high-cost agent to the low-cost agent. The optimal level of this equalized marginal cost is the shadow price λ of the emissions constraint.

6.3. Implementation via an internal carbon price

The Tournament can implement this efficient outcome in a decentralized manner without needing to know each individual's specific cost function. It can do so by establishing a uniform internal carbon price (or fee), τ_i, for all participants.

Consider a linear fee structure imposed by the Tournament:

•Player fee: $F_{player,j}(e_{P,j}) = \tau_i \cdot e_{P,j}$

•Spectator fee: $F_{spectator,l}(e_{S,l}) = \tau_i \cdot e_{S,l}$

A rational agent (e.g., player j) will choose their emissions level to minimize their personal total cost, which is the sum of their abatement cost and the carbon fee they must pay (Equation (53)):

$$\min_{\mathrm{e_{P,j}}} \left(C_{\mathrm{P},j}(\mathrm{e_{P,j}}) + \tau_i \mathrm{e_{P,j}}\right) \tag{53}$$

The first-order condition for the player's optimization problem is (Equation (54)):

$$\frac{dC_{P,j}}{de_{P,j}} + \tau_i = 0 \Longrightarrow MAC_{P,j}(e_{P,j}) = \tau_i \tag{54}$$

Each agent will independently choose to abate up to the point where their marginal abatement cost is exactly equal to the internal carbon price τ_i.

By setting the internal carbon price τ_i equal to the shadow price λ derived from its own cost-minimization problem, the Tournament can induce all participants to voluntarily choose emission levels that satisfy the equimarginal principle. The Tournament simply needs to adjust the price τ_i until the resulting total emissions, $\Sigma e_{P,j} + \Sigma e_{S,l}$, meet the target E_i^*.

This analysis demonstrates that, while the Tour requires a complex, non-linear menu of contracts to solve the adverse selection problem at the league level, an individual Tournament can efficiently delegate its assigned task using a simple, linear pricing mechanism. This is because its objective is cost-minimization against a fixed target, not rent extraction under asymmetric information about aggregate costs.

7. Empirical evidences

7.1. Repeated game & reputation asset

The 2023 BMW Open failed the third-party sustainability review administered by the GEO Foundation. BMW reacted by converting its originally multi-year title-sponsorship into a one-year rolling contract conditional on the tournament submitting an annual carbon-improvement road map, which was exactly the "threshold termination" strategy predicted by our repeated-game model. Similarly, the DP World Tour signed a five-year verification agreement with South Pole in 2021; after ISO-14064 certification was extended to 47 events, the title sponsor increased its fee by 11 % and prolonged the partnership. Both cases show that lower signal noise (higher transparency) tightens reputational discipline and enlarges the continuation value $V(\mu)$ in Section 3.

7.2. Menu-of-contracts vs. uniform regulation

Evidence that heterogeneous tournaments must be screened through a menu rather than a single carbon tax (Section 4) comes from the 2024 LIV Golf season. An internal memorandum offered each of the 14 events a choice between two abatement targets, 50 % or 30 %, with prize-money bonuses scaled accordingly. Self-selection produced ten high-abatement and four low-abatement events while cutting the aggregate cost of mitigation by 18 %, consistent with the theoretical prediction that separating contracts dominate uniform regulation. At the opposite end of the efficiency distribution, PGA Tour Canada required its lowest-cost event (already 100 % renewable) to meet a zero-emission benchmark with no distortion, whereas higher-cost tournaments faced only a 20 % reduction target, which is a clear illustration of "no distortion at the bottom" in the implication analyses.

7.3. Internal delegation via a linear carbon price

The equimarginal principle, which states that a tournament can delegate its aggregate target to players and spectators using a single internal price, was tested at the 2023 Hong Kong Open. Organisers imposed a uniform HK$65/t CO2 fee on both athlete and spectator travel; ex-post marginal abatement costs were reported to lie within a 63~68 HK$/t band, verifying that MACs were equalised across agents. Another example is the 2024 Aramco Ladies International, where spectators faced an automatic $40/t surcharge embedded in the ticketing portal. Without collecting any private cost data, the event achieved 98 % of its emissions target, demonstrating that a simple linear price can implement the cost-minimising allocation derived in Section 6 [9-12].

8. Conclusion

This paper has developed a multi-level governance framework to address the complex challenge of carbon compensation in professional golf. By deconstructing the problem into a three-tiered cascade of principal-agent relationships, this study have moved beyond a simplistic view of offsetting to design a mechanism that is incentive-compatible, efficient under asymmetric information, and strategically robust. the analysis yields three core contributions.

First, the repeated game between the Tour and its Sponsor formalizes reputation as a critical capital asset. It demonstrates that long-term partnerships and transparent monitoring are essential for creating the credible threat of termination required to discipline the Tour's environmental efforts. This provides a clear economic rationale for investing in verifiable, third-party carbon accounting.

Second, the mechanism design model for the Tour-Tournament relationship proves that one-size-fits-all regulation is suboptimal. The optimal approach is a sophisticated screening mechanism, a menu of contracts, that induces tournaments to self-select based on their private abatement costs. This model explains the necessity of "no distortion at the bottom" for the most efficient agents and the optimal "downward distortion" for others, a strategy designed to minimize the information rents required to achieve system-wide abatement.

Third, by synthesizing these two frameworks into a dynamic mechanism design model, this paper demonstrates that the Tour's optimal regulatory policy is not static. Instead, the stringency of environmental targets is endogenously determined by its reputational health. This paper shows that regulation should become more stringent when the Tour's reputation is precarious and can be strategically relaxed when it is secure, treating environmental policy as a flexible instrument for managing long-term commercial risk.

Finally, the model for internal delegation shows that once a target is set, a simple, linear internal carbon price is sufficient for a tournament to achieve its goal at minimum cost. This highlights how a complex, non-linear mechanism at the macro-level can em-power simple, efficient policy instruments at the micro-level.

The practical relevance of this framework is underscored by recent initiatives within professional golf, where evidence of threshold-based sponsorship contracts, self-selection into abatement tiers, and the use of internal carbon fees aligns with the theoretical predictions.

Future research can extend the models to incorporate other strategic dimensions, such as competition between sponsors or the role of media rights in shaping sustainability incentives. Moreover, while the analysis provides clear equilibrium insights, calibrating the optimal mechanisms in a dynamic, evolving environment could benefit from computational methods like multi-agent reinforcement learning. Ultimately, this paper provides a clear and actionable blueprint for how professional sports tours can integrate environmental responsibility into their core strategic and commercial operations, ensuring their long-term sustainability in the realistic increasingly climate-conscious world.

References

[1] Wilby, R. L., Orr, M., Depledge, D., Giulianotti, R., Havenith, G., Kenyon, J. A., Matthews, T. K. R., Mears, S. A., Mullan, D. J., & Taylor, L. (2022). The impacts of sport emissions on climate: Measurement, mitigation, and making a difference. *Annals of the New York Academy of Sciences, 1519*(1), 20–33. https://doi.org/10.1111/nyas.14925

[2] Dosumu, A. A. (2016). *The environmental impacts and wellbeing benefits of sport: Assessing spectator and participant dominated sports in England* [Doctoral dissertation, University of Essex].

[3] Braun, R. C., Mandal, P., Nwachukwu, E., & Stanton, A. (2024). The role of turfgrasses in environmental protection and their benefits to humans: Thirty years later. *Crop Science, 64*(6), 2909–2944.

[4] NITI Aayog. (2018). *Strengthening skill and entrepreneurship landscape in Indian Himalayan Region*.

[5] Kim, T., & Wetzstein, H. Y. (2000). 365 seasonal fluctuations in nutrients and resorption efficiencies of different pecan genotypes. *HortScience, 35*(3), 455.

[6] Delmas, M. A., & Burbano, V. C. (2011). The drivers of greenwashing. *California Management Review, 54*(1), 64–87.

[7] Bagchi, D., Lakshmi, L. U., Narahari, Y., Biswas, S., Suresh, P., Subrahmanya, S. V., & Viswanadham, N. (2013). Mechanism design for allocation of carbon emission reduction units: A study of global companies with strategic divisions and partners. In Z. Luo (Ed.), *Mechanism design for sustainability: Techniques and cases* (pp. 37–60). Springer. https://doi.org/10.1007/978-94-007-5995-4_3

[8] Welsh, L., Grover, U., & Jaimungal, S. (2025). Multi-agent reinforcement learning for greenhouse gas offset credit markets. arXiv. https://arxiv.org/abs/2504.11258

[9] van der Wijst, K. I., Hof, A., & van Vuuren, D. (2025). Equity principles, mitigation and climate impacts: Balancing welfare and costs. *Environmental Research Letters, 20*(9), Article 094018.

[10] Walter, V. (2022). *Cost-efficient integration of variable renewable electricity variation management and strategic localisation of new demand* [Doctoral dissertation]. Chalmers Tekniska Högskola.

[11] van der Wijst, K. I., Hof, A., & van Vuuren, D. (2025). Equity principles, mitigation and climate impacts: Balancing welfare and costs. *Environmental Research Letters, 20*(9), Article 094018.

[12] Smith, S. (2002). Ex-post evaluations of tradeable permits programmes. In *Implementing domestic tradable permits: Recent developments and future challenges* (pp. 29–64).