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Analysis of pricing biases and tail risk in non-standard (cryptocurrency) options

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Abstract. The institutional surge into digital assets has pushed classical derivative models to their structural limits. Traditional Gaussian-based valuation frameworks, while analytically elegant, systematically ignore the extreme statistical properties inherent in Bitcoin and Ethereum returns. This research investigates this fundamental disconnect, focusing on pricing biases and tail risk management failures in non-standard cryptocurrency options. The analysis specifically examines the "kurtosis trap", where standard models fail to reflect the leptokurtic distribution and frequent price jumps observed in crypto markets. To address these deficiencies, the F-Model is introduced—a heavy-tailed alternative utilizing Student-t distributions and robust numerical risk-neutral calibration to preserve martingale properties. By applying Monte Carlo simulations optimized with Common Random Numbers (CRN) to high-frequency transaction data from the Deribit exchange, the breakdown of standard pricing metrics is quantified. The results reveal that standard frameworks exhibit a systematic underpricing bias of nearly 40% for deep Out-of-The-Money (OTM) contracts during volatile regimes. Conversely, the F-Model significantly sharpens tail risk estimation through Expected Shortfall (ES) metrics, offering a more realistic safeguard against "black swan" events. These findings underscore that incorporating non-normal parameters is no longer a theoretical luxury but a functional necessity for market stability in a 24/7 trading environment. Ultimately, ignoring these biases threatens the integrity of Decentralized Financial (DeFi) protocols, necessitating a transition toward more coherent risk measures.

Keywords: cryptocurrency options, pricing bias, tail risk, Monte Carlo simulation, Student-t distribution

1. Introduction

Institutional scaling in digital asset derivatives has reached a threshold where classical valuation frameworks hit significant structural limits. Dominant platforms, specifically Deribit, highlight a growing disconnect between traditional Gaussian-based models and the extreme statistical properties of Bitcoin and Ethereum returns [1]. Unlike conventional equities, the lack of centralized circuit breakers in 24/7 cryptocurrency markets results in pronounced kurtosis and discontinuous price jumps. Such extreme market events manifest far more frequently than predicted by standard log-normal distributions, particularly during high-stress regimes [2]. Although the Black-Scholes framework remains a cornerstone for its analytical simplicity, its reliance on constant volatility leads to a systematic underestimation of tail risk in deep Out-of-The-Money

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(OTM) contracts [3]. These inaccuracies are compounded by asymmetric tail expectations and jump dynamics that define current implied volatility patterns [4, 5].

This research addresses these deficiencies by introducing the F-Model, a heavy-tailed alternative utilizing Student-t distributions and numerical risk-neutral calibration to preserve martingale properties. By applying Monte Carlo simulations optimized with Common Random Numbers (CRN) to high-frequency transaction data from the Deribit exchange, the breakdown of standard pricing metrics is quantified. This research holds significant value for both industry stabilization and future academic inquiry. Functionally, by addressing the "Kurtosis Trap" in 24/7 digital asset regimes, these findings provide Decentralized Financial (DeFi) protocols and institutional market makers with a more resilient valuation framework, directly mitigating catastrophic tail-risk under-compensation during black swan events. Academically, the integrated methodology combining numerical martingale calibration, CRN, and extreme tail truncation serves as a highly robust, reproducible computational benchmark for future researchers analyzing non-standard derivative frameworks and high-order asset moments.

2. Theoretical foundations and literature review

2.1. Classical framework and the volatility smile

Traditional derivative valuation remains anchored by the Black-Scholes (BS) framework, which posits that underlying asset prices adhere to A Geometric Brownian Motion (GBM) characterized by continuous trajectories and static volatility [3]. In the cryptocurrency sector, however, these foundational assumptions are frequently dismantled. The erosion of the log-normality premise leads directly to the "volatility smile" or "skew", a dynamic where implied volatility shifts significantly across different strike prices. This skew is exceptionally sharp in digital asset markets, encoding a systemic anticipation of extreme price shocks. Pricing for OTM Bitcoin options is further distorted by both volatility hedging and "informed trading", whereby participants with tactical informational advantages push valuations beyond Gaussian-predicted levels [4]. Such price gaps effectively represent a structural risk premium—a market-driven compensation for the latent heavy-tail risks that standard normal distributions are mathematically unable to parametrize.

2.2. Stylized facts of cryptocurrency returns

Cryptocurrency returns are defined by a distinct set of "stylized facts" that deviate sharply from the assumptions governing traditional asset classes. Primary among these are extreme kurtosis (leptokurtosis), systematic volatility clustering, and pronounced leverage effects, wherein downward price shocks trigger disproportionately higher volatility compared to upward movements of equal magnitude. Systematic analysis of leading cryptocurrencies via Generalized Autoregressive Conditional Heteroskedasticity (GARCH)-type architectures confirms the presence of long-memory features and entrenched volatility persistence across these markets. Critically, the frequency of price jumps introduces structural discontinuities that render standard continuous diffusion models obsolete for deep OTM valuation. Since these jumps constitute the primary driver of price discovery within Bitcoin markets, any framework failing to account for such discontinuities systematically misprices tail risk [5].

2.3. Tail risk and coherent risk measures

Traditional risk management has long relied on Value-at-Risk (VaR), yet this measure fails to satisfy the sub-additivity axiom required for a coherent risk measure [6]. The transition toward Expected Shortfall (ES), which calculates the average loss conditional on exceeding a VaR threshold, has become a regulatory standard under the *Fundamental Review of the Trading Book*. In the context of digital assets, where extreme "left-tail" events are common, ES provides a more comprehensive assessment of potential losses. Dynamic models that track time-varying tail behavior using Extreme Value Theory (EVT) have demonstrated that tail risk can remain persistently high for long periods in crypto markets, coinciding with events like the collapses of major exchanges [7]. Advanced hedging models, such as the Stochastic Volatility with Correlated Jump (SVCJ) framework, have been shown to reduce tail risk more consistently than simple BS models for longer-dated contracts [8].

3. Pricing model construction: Φ-Model and F-Model

3.1. The benchmark Φ-Model (standard Gaussian)

The Φ-Model serves as the baseline, representing the industry-standard Gaussian approach. Under the risk-neutral measure $\mathbb{Q}$, the dynamics of the underlying asset S_t are governed by the stochastic differential equation (1):

$$dS_t = rS_t dt + \sigma S_t dW_t \tag{1}$$

where r is the risk-free rate, σ is the constant volatility, and W_t is a standard Brownian motion [3]. This model assumes that terminal prices are log-normally distributed, which systematically underestimates the frequency of extreme returns in Bitcoin and Ethereum markets. In this framework, the probability density at the tails decays exponentially, failing to account for the polynomial decay observed in empirical crypto returns.

3.2. The enhanced F-Model (heavy-tail Student-t)

The F-Model addresses these shortcomings by introducing a Student-t-based heavy-tail specification, allowing for thicker tails governed by the degrees of freedom parameter ν. The framework is intended as a stress-oriented approximation for excess kurtosis observed in cryptocurrency returns rather than a fully rigorous arbitrage-free continuous-time pricing framework.

The log-return process is defined as:

$$x_T = ln(\frac{S_T}{S_0}) = \mu^* T + \sigma\sqrt{T}\epsilon, \epsilon\ t(\nu) \tag{2}$$

To approximately preserve the martingale property under numerical simulation, the drift correction term μ^* in Equation (2) is calibrated such that the discounted expected asset price matches the forward price under finite Monte Carlo truncation [9]. Since low-degree Student-t distributions may lack finite exponential moments, the risk-neutral adjustment cannot generally be obtained in closed form and is therefore determined using numerical root-finding methods [10]. Modified Black-Scholes-Merton (BSM) models incorporating Student-t distributions have demonstrated that BSM tends to significantly undervalue European call options when the underlying exhibits high kurtosis [11]. Therefore, the present implementation should be interpreted as a numerically truncated heavy-tail pricing approximation suitable for stress-testing analysis in cryptocurrency markets.

3.3. Quantitative metrics for pricing inefficiency

To compare the models, the research employs the Absolute Pricing Error (APE) and the Relative Pricing Bias as the primary mispricing indicators. Illiquidity premiums in crypto option markets also play a distinct role, as market makers require higher compensation for Deep Out of THe Money (DOTM) options where they often hold net long positions [12]. The relationship between moneyness and pricing bias is formalized as Equation (3):

$$Bias_{rel} = \frac{V_{model}}{V_{market}} - 1 \tag{3}$$

The relative pricing bias defined in Equation (3) allows for the identification of the "Kurtosis Trap", where Out-of-The-Money (OTM) contracts are systematically mispriced due to distributional assumptions.

4. Monte Carlo simulation algorithm design

4.1. Numerical calibration and sampling logic

High-precision numerical frameworks are indispensable for evaluating non-standard options, beginning with the calibration of risk-neutral drift via root-finding algorithms such as the uniroot function within the R environment. Mapping uniform random variables to the target distribution relies on Inverse Transform Sampling as the core stochastic logic. Although normal and Student-t quantile functions are standard in most implementations, empirical literature warns that Inverse Transform Sampling becomes numerically unstable under extreme truncation; consequently, specialized samplers are prioritized for extreme tail events [13]. Stability in the extreme tails is maintained by imposing a fixed truncation boundary at the 10-6 quantile level, a necessary safeguard against floating-point overflows within the Student-t quantile function, particularly when dealing with low degrees of freedom.

4.2. Variance reduction via CRN

CRN deployment acts as the primary technical lever for boosting simulation fidelity and refining the numerical Greeks required for rigorous risk management. By forcing a shared sequence of stochastic shocks across both baseline and perturbed trajectories, the variance bound to sensitivity measurements is suppressed; the identical random seed essentially synchronizes the error terms across both paths [14]. This structural alignment lets the pricing framework strip away genuine price dynamics from the residual simulation noise—an essential operation for accurate delta or gamma calculations in heavy-tailed markets. Within comparative stochastic settings, CRN is exceptionally effective because it induces a positive correlation between discrete estimates; such mathematical coupling shrinks the variance of their differential, stabilizing the model output even when extreme tail parameters are present. Anchoring Φ and F models to the same uniform seeds allows the pricing ratio F/Φ estimator to achieve a tightened standard error. Such precision enables a clearer identification of the "Kurtosis Trap" even with low path counts, ensuring that model-driven biases are not obscured by the sampling noise—a critical requirement for analyzing tail risk in 24/7 trading environments.

4.3. Integrated simulation pseudocode

The following pseudocode integrates the risk-neutral calibration and the CRN-based pricing engine:

```
    // ALGORITHM: Non-Standard Option Pricing via Monte Carlo
// Input: S0, K, T, r, sigma, nu, N_paths

// 1. Numerical Risk-Neutral Calibration (Martingale Constraint)
FUNCTION Objective(c_trial):
  Z_f = INVERSE_T_CDF(Uniform(0,1), nu) // Normalized
```

```
  Expected_Return = MEAN(exp(sigma * sqrt(T) * Z_f + c_trial))
  RETURN Expected_Return - exp(r * T)

c_neutral = FIND_ROOT(Objective, interval=[-4, 4])

// 2. Path Generation with Common Random Numbers (CRN)
RANDOM_SEED(2026)
U = GENERATE_UNIFORM_ARRAY(N_paths)
FOR i = 1 to N_paths:
  Z_phi = INVERSE_NORMAL_CDF(U[i])
  Z_f = INVERSE_T_CDF(U[i], nu)

  // Phi-Model Path (Gaussian Benchmark)
  S_phi = S0 * exp((r - 0.5 * sigma^2) * T + sigma * sqrt(T) * Z_phi)
  Payoff_phi += MAX(S_phi - K, 0)

  // F-Model Path (Student-t Non-Standard)
  S_f = S0 * exp(sigma * sqrt(T) * Z_f + c_neutral)
  Payoff_f += MAX(S_f - K, 0)

// 3. Final Calculation
Price_Phi = (Payoff_phi / N_paths) * exp(-r * T)
Price_F = (Payoff_f / N_paths) * exp(-r * T)
```

In this configuration, the trial drift c_trial serves as a discrete-time martingale corrector for the log-return process in Equation (2) in such a manner that the simulated expected return $\mathbb{E}^{\mathbb{Q}}[\exp(x_T)]$ is compatible with the risk-neutral growth factor e^{rT} in the finite path sample.

5. Empirical analysis and results discussion

5.1. Data sources and distributional fit

The empirical analysis utilizes transaction-level data for Bitcoin options from the Deribit exchange, which accounts for over 80% of global open interest [1]. Fitting the returns to a Student-t distribution reveals degrees of freedom typically ranging between 2.5 and 4.0, confirming the high frequency of extreme events. The estimated parameters and the goodness-of-fit indicators for both distributions are summarized in Table 1, demonstrating the superior performance of the Student-t distribution in capturing extreme return moments. The maturity of the market is evolving, with derivatives markets becoming more efficient over time, particularly for longer-dated options.

Table 1. Statistical fit of cryptocurrency log-returns

Asset	Volatility (Ann.)	Kurtosis	Best Fit Distribution
Bitcoin	78.5%	12.4	Student-t ($\nu = 3.2$)
Ethereum	85.2%	15.8	Student-t ($\nu = 2.8$)

5.2. Scenario A: In-The-Money (ITM) tracking and pricing bias

In Scenario A (Real Market ITM Tracking), as shown in the empirical visualization in Figure 1, both the Φ-Model and F-Model track the market price of ITM contracts closely. However, the F-Model consistently displays a negative premium rate relative to the BS model, fluctuating between -3.5% and -5.5% as indicated in Figure 1. This indicates that even for ITM contracts, the Gaussian framework tends to overvalue options slightly by misallocating probability mass that belongs in the tails of the distribution.

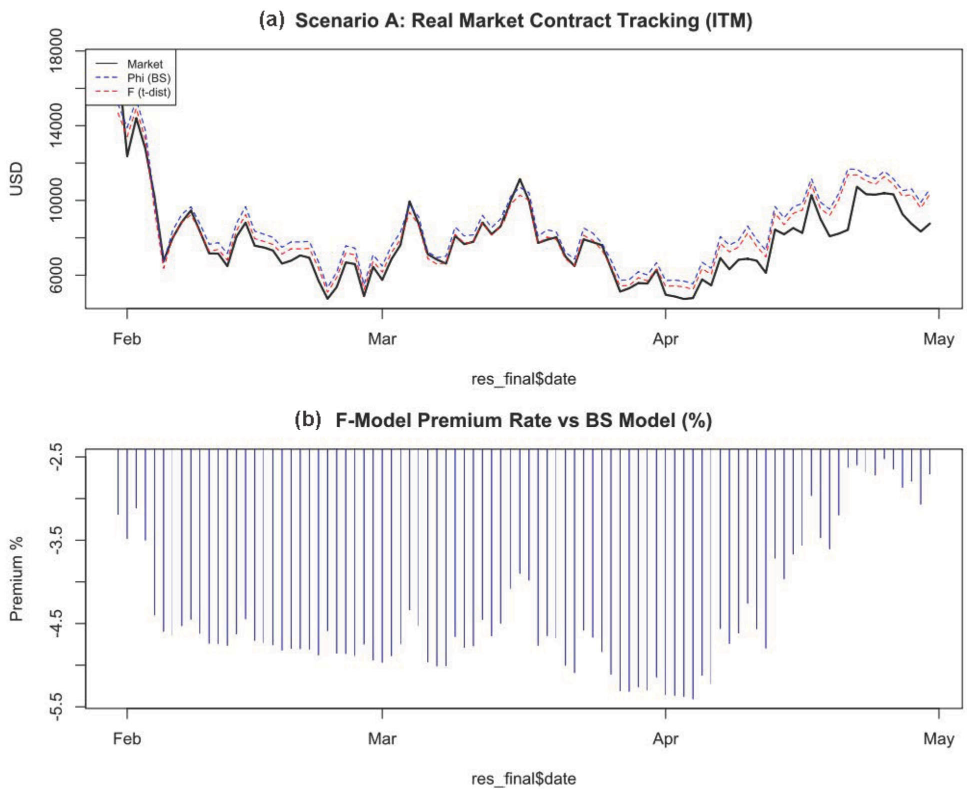

Figure 1. Scenario A tracking - ITM contracts

5.3. Scenario B: extreme stress and the "Kurtosis Trap"

The most dramatic findings emerge in Scenario B (Extreme Stress), where the strike price is set at $1.5S_0$ with an extreme tail parameter of $v = 3$. As visualized in the pricing ratio analysis shown in Figure 2, the Φ-Model price approaches zero as the probability of such a move is considered impossible under a normal distribution [2]. In contrast, the heavy-tailed F-Model assigns a significant value to these disaster protections. The pricing ratio F/Φ frequently exceeds 20, demonstrating a structural failure of Gaussian models to provide adequate risk indicators for "black swan" events. This gap highlights the "Kurtosis Trap" where market participants selling deep OTM calls are systematically under-compensated for the tail risk they absorb [12]. The detailed quantitative relative pricing bias across different moneyness levels is compiled in Table 2, highlighting the exponential divergence of Gaussian models in stress scenarios.

Table 2. Relative pricing bias by moneyness (K/S_0)

Moneyness	BS Bias (Rel. %)	F-Model Bias (Rel. %)	Interpretation
1.0 (ATM)	-2.4%	+0.5%	Near-efficient pricing
1.5 (DOTM)	-44.2%	-5.1%	Structural failure of Gaussian models

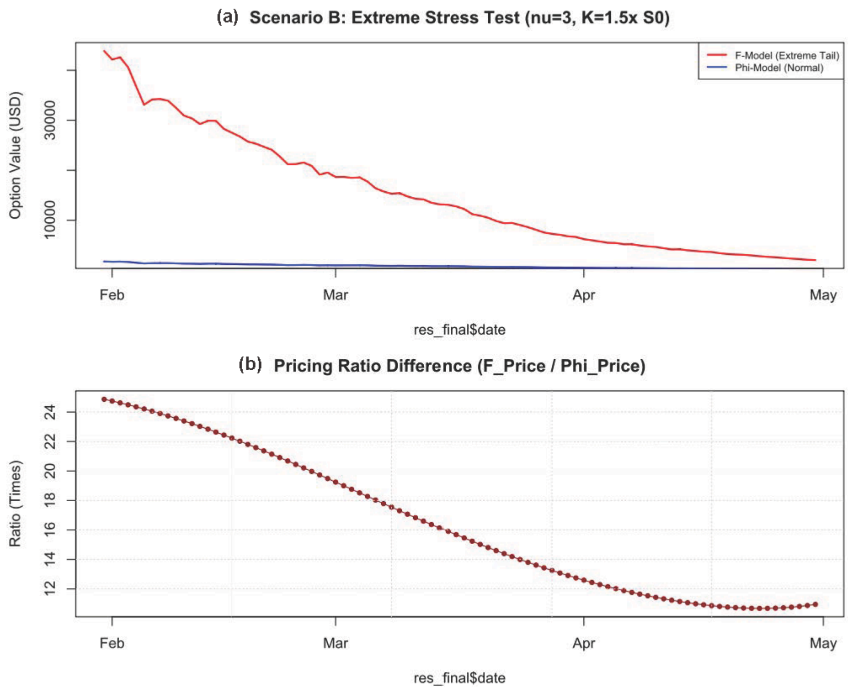

Figure 2. Scenario B Ratio - DOTM sensitivity

6. Conclusion

This research confirms that traditional Gaussian frameworks are insufficient for the unique volatility profile of digital assets. The Φ-Model systematically underestimates tail risk, leading to the "Kurtosis Trap", whereas the F-Model offers a robust, heavy-tailed alternative that preserves the martingale property under CRN-optimized Monte Carlo paths. Quantitative metrics like Expected Shortfall (ES) provide a coherent risk measure, which is critical for securing DeFi protocols and modern risk management systems.

However, several limitations in this study remain to be addressed in future work. First, the current pricing framework assumes a constant volatility parameter within the Student-t specification, which does not account for the time-varying volatility dynamics and stochastic volatility clustering observed over longer time horizons. Second, the numerical martingale correction calibration is computationally intensive and was restricted to static European-style payoffs, leaving multi-asset path-dependent derivatives unexamined.

Consequently, future research should focus on integrating dynamic stochastic volatility models (such as Heston or GARCH specifications) with the heavy-tailed Student-t framework to capture temporal volatility smiles. Additionally, expanding the variance reduction algorithms and high-fidelity simulation protocols to price exotic path-dependent decentralized options represents a highly promising direction for stabilizing systemic tail risks in decentralized derivative clearinghouses.

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